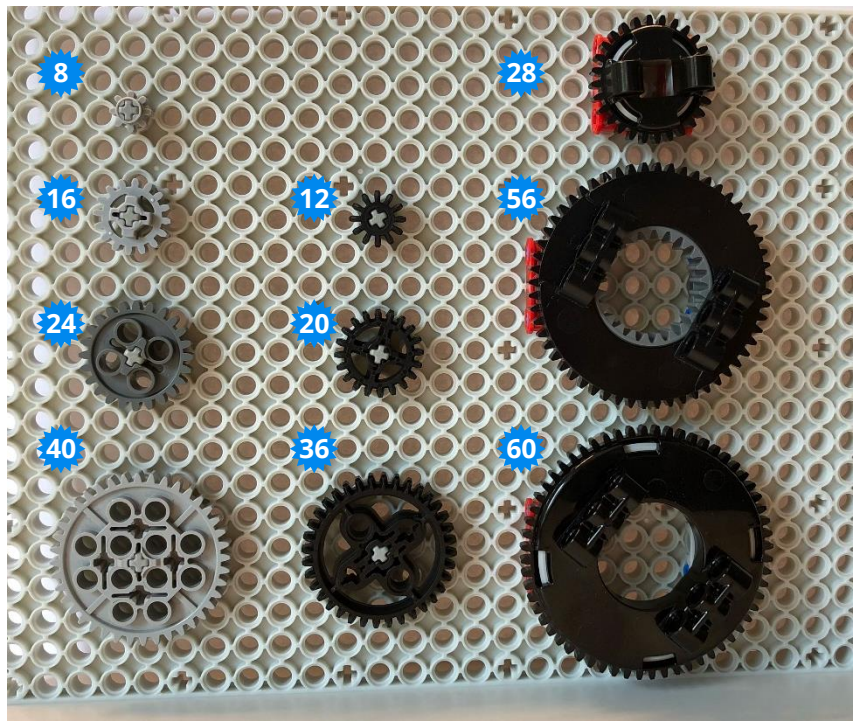


LEGO Technic Pairs of Gears



LEGO Technic - Gears and the magic number 16

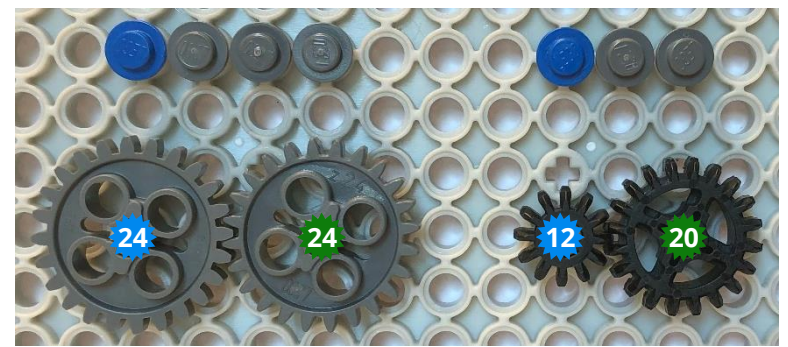
All gears available (Aug 19, 2018) and their number of teeth.



If the total of all teeth of a pair of gears is a multiple of 16, the two gears can be put into a straight line.

The distance between the axes is the total of all teeth divided by 16.

$$24+24 = 48 \rightarrow \text{distance } 3$$



$$12+20 = 32 \rightarrow \text{distance } 2$$



A two-dimensional grid provides even more opportunities for a pair of gears

The "Theorem of Pythagoras" to calculate the distance in the grid.

		a = 4				
	1,00	2,00	3,00	4,00	5,00	6,00
c = ?	1,41	2,24	3,16	4,12	5,10	6,08
		2,83	3,61	4,47	5,39	6,32
			4,24	5,00	5,83	6,71
				5,66	6,40	7,21
					7,07	7,81
						8,49

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 c &= \sqrt{a^2 + b^2} \\
 &= \sqrt{4^2 + 2^2} \\
 &= \sqrt{16 + 4} \\
 &= \sqrt{20} = 4,47
 \end{aligned}$$

The distances multiplied by 16 and rounded-up to the next whole number, provides additional numbers for "the total of all teeth".

		16	32	48	64	80	96
		23	36	51	66	82	98
			46	58	72	87	102
				68	80	94	108
					91	103	116
						114	125
							136

Values marked red cannot be reached with two gears.



All possible gear pairs on the grid in one view

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- Number of teeth of upper-left gear
- Number of teeth of matching gear

Example **24** and **12**

Why is 16 the magic number?

For all gears the width (x) of each tooth has to be the same to allow for any combination of gears.

Therefore, the circumference (U) of a gear with n teeth is $U = x * n$

In general the circumference of a circle is: $U = 2\pi * r$

$$x * n = 2\pi * r$$

The smallest gear has 8 teeth and a diameter of 1 LEGO unit:

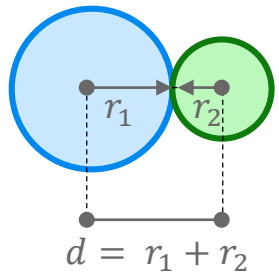
$$x * 8 = 2\pi * \frac{1}{2}$$
$$x = \frac{\pi}{8}$$



Consequently for all gears:

$$\frac{\pi}{8} * n = 2\pi * r$$

$$r = \frac{n}{16}$$



For two circles to touch each other the distance d between the two center points needs to be the sum of their radiuses.

Combining both yields:

$$d = r_1 + r_2 = \frac{n_1}{16} + \frac{n_2}{16} = \frac{n_1 + n_2}{16}$$

The distance between the axes is determined by the total of all teeth divided by 16.